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Fractal Geometry is not the Geometry of Nature

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Abstract—In recent years the magnificent world of fractals has been revealed. Some of the fractal images resemble natural forms so closely that Benoit Mandelbrot's hypothesis, that the fractal geometry is the geometry of natural objects, has been accepted by scientists and non-scientists alike. The present paper critically examines Mandelbrot's hypothesis. It first analyzes the concept of a fractal. The analysis reveals that fractals are endless geometrical processes, and not geometrical forms. A comparison between fractals and irrational numbers shows that the former are ontologically and epistemologically even more problematic than the latter. Therefore, it is argued, a proper understanding of the concept of fractal is inconsistent with ascribing a fractal structure to natural objects. Moreover, it is shown that, empirically, the so-called fractal images disconfirm Mandelbrot's hypothesis. It is conceded that the fractal geometry can be used as a useful rough approximation, but this fact has no bearing on the physical theory of natural forms.

1. Mandelbrot's Hypothesis Concerning the Fractal Geometry of Nature

In recent years the magnificent world of fractals has been revealed. Some of the fractal images closely resemble natural forms: flora, fauna and landscapes.[1] The resemblance is so great, that it seems it might be possible to explain the origin or causes of such forms in terms of fractals. Indeed, Benoit Mandelbrot called his book, in which fractals resembling nature were first presented, *The Fractal Geometry of Nature*. Mandelbrot advocates the hypothesis that numerous natural forms are fractals and, therefore, are to be described and analyzed by the fractal geometry. This geometry, he argues, is the appropriate one for this matter, and not the traditional non-fractal geometries, Euclidean and others.[2] Mandelbrot's hypothesis has caught on among scientists, including those specializing in the subject, and thence also among

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[1]Numerous examples are found in the literature. See for example Benoit Mandelbrot (1983) *The Fractal Geometry of Nature* (New York: W.H. Freeman) and H. O. Peitgen and D. Saupe, eds (1988), *The Science of Fractal Images* (New York: Springer).
[2]Mandelbrot, in *The Fractal Geometry of Nature*, calls these geometries 'Euclid'. This name may be misleading, as it also refers to non-Euclidean geometries.

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non-scientists.[3] The prevailing view of the matter, as expressed by Jurgens, Peitgen and Saupe is, that

> Fractals are much more than a mathematical curiosity. They offer an extremely compact method for describing objects and formations . . . Fractal geometry seems to describe natural shapes and forms more gracefully and succinctly than does Euclidean geometry.[4]

However, in spite of the great visual impact of fractal images, Mandelbrot's hypothesis is far from being a satisfactory scientific theory. In fact, with regard to natural forms, the famous images are all it has to offer at present. That this is the present-day situation is well known. The interesting question is, whether this hypothesis has the potential to become a scientific theory. Many think it does; the present paper challenges this view. Leo P. Kadanoff expressed the problematic state of the present day 'science of fractals', when he noted that

> . . . further progress in this field depends upon establishing a more substantial theoretical base, in which geometrical form is deduced from the mechanisms that produce it. Lacking such a base, one cannot define very sharply what types of questions might have interesting answers . . . Without that underpinning much of the work on fractals seems somewhat superficial and even slightly pointless. It is easy, too easy, to perform computer simulations upon all kinds of models and to compare the results with each other and with real world outcomes. But without organizing principles, the field tends to decay into a zoology of interesting specimens and facile classifications. Despite the beauty and elegance of the phenomenological observations upon which the field is based, the physics of fractals is, in many ways, a subject waiting to be born.[5]

But Kadanoff does not express doubt regarding the very plausibility of Mandelbrot's hypothesis. He too seems to believe it has a potential for becoming what he calls 'the physics of fractals'.[6] The present paper critically examines both Mandelbrot's hypothesis that nature has a fractal geometry, and the belief expressed by Kadanoff that there is a physics of fractals waiting to be born. Let me be more precise. Mandelbrot's hypothesis is that fractals are to be found among the spatial forms of natural objects, as well as among temporal properties of natural systems (for example, frequencies) and spatio-temporal dispositions (i.e. initial conditions of chaotic systems). I wish to argue that the hypothesis is basically wrong in its spatial part. Mandelbrot may have a point regarding the temporal parts of his hypothesis, but

[3]For an example of scientists' view of the matter see the sayings of R. F. Voss and M. F. Barnsley in Peitgen and Saupe, *The Science of Fractal Images* p. 21 and pp. 219–220. An instance of a non-scientific use of Mandelbrot's hypothesis is found in Larry Short (1991) 'The Aesthetic Value of Fractal Images', *British Journal of Aesthetics* **31,** 342. Short sees nature as trivially having a fractal geometry. In this he follows Mandelbrot's opinion in Benoit Mandelbrot (1981) 'Scalebound and Scaling Shapes: a Useful Distinction in the Visual Arts and in the Natural Sciences', *Leonardo* **14,** 45.

[4]H. Jurgens, H. O. Peitgen and D. Saupe (1990) 'The Language of Fractals', *Scientific American* **263,** 40.

[5]Leo P. Kadanoff, (1986) 'Fractals: Where's the Physics?', *Physics Today*, February, **6,** quoted from p. 7.

[6]Mandelbrot himself, however, saw in Kadanoff's words a heresy. See B. Mandelbrot (1986) 'Multifractals and Fractals', *Physics Today*, September, 11–12.

this I leave for a separate discussion. The theory regarding chaotic systems makes use of fractals having a temporal element, as they are embedded in the phase space. The success of this scientific domain has had the unfortunate side-effect of causing many to believe that Mandelbrot's hypothesis concerning the spatial properties of natural objects is as meaningful and as successful. This mistake I wish to correct presently. (Hereafter, when referring to Mandelbrot's hypothesis I will mean its spatial part.)

The criticism regarding the spatial part of Mandelbrot's hypothesis rests on four main arguments. First, I wish to propose that fractals are geometrically abnormal, in the sense that they are better viewed as geometrical processes, not geometrical objects. In this respect, they are ontologically and epistemologically even more problematic than irrational numbers. Therefore, the claim that spatial forms of natural objects are fractals is inherently contradictory. Second, Mandelbrot's hypothesis is empirically mistaken. It is based on the so-called fractal images. But some of these are generated by functions whose very classification as fractals is mistaken. Others are unnatural because they are embedded in essentially non-isotropic abstract spaces. As to the rest, their success in imitating nature can be better explained without resorting to fractal geometry, and therefore has nothing to do with fractals. Third, fractal geometry can be used as a useful approximation of natural spatial forms in some cases. But, if it turns out that a system and its approximation are dominated by different scientific principles, the latter's nature should not be taken as an indication of the former's nature. Finally, as Kadanoff pointed out, Mandelbrot's hypothesis regarding spatial forms is a mere collection of abstract functions which are not even serious candidates for being part of a science, since they lack interpretation. Such an interpretation cannot be found within known science, as the hypothesis is inconsistent with the atomistic paradigm. Alternative, radically non-atomistic interpretations have not been offered, even in the most general terms.

Let us discuss these four arguments, starting with an examination of the concept of a fractal.

2. Fractals As Geometrical Processes

The name 'fractal', coined by Mandelbrot, denotes a geometrical kind, not unlike 'line'. The concept of a fractal cannot be defined in terms of basic concepts such as point, line, etc., whether their interpretation is Euclidean or otherwise, and in this sense the fractal is a geometrical type in its own right, having the same status as the line. Fractal geometry has been developed to deal with these unique geometrical entities.[7] The set of fractals can be divided into several families, as can the set of lines.

[7]For details of this geometry, see K. Falconer (1990) *Fractal Geometry* (New York: Wiley).

The famous Mandelbrot set belongs to one of them.[8] The forms belonging to the different families of fractals may seem at first sight very different from each other, but they share important properties, and these make them a single geometrical kind. As Mandelbrot's hypothesis relates to all the sorts of fractals, the concept of a fractal has to be examined in the most general terms.

The definition of a line is hard to grasp, since pure length without width and depth is impossible to visualize. But one can overcome this difficulty by viewing the concept of a line as a limit concept. The grasping of fractals is a far harder case. In order to appreciate the special difficulties fractals pose, it is necessary to turn first to their definition.

For a reason given below, I choose to define fractals, or see the necessary and sufficient condition for being a fractal, as objects having infinitely many details within a finite volume of the embedding space.[9] Consequently, fractals have details on arbitrarily small scales, revealed as they are magnified or approached. Let us call this trait 'infinite complexity'.[10]

While each of the various kinds of fractals is mathematically well defined, there is not yet a generally accepted definition of the general concept of a fractal. However, there is general agreement regarding which forms belong to the set of fractals and which do not, with problematic marginal cases. Therefore one may offer a definition of fractals, as long as it accords with this general agreement, but one is obliged to justify the offered definition.[11] It seems to me desirable that the definition should establish and emphasize the dichotomy between forms that are and are not analyzable by fractal geometry, for the following reasons. The application of fractal geometry to finitely complex forms, i.e., forms having a finite number of details, is worse than superfluous. Analysis of such forms in terms of non-fractal geometries is more exact and necessitates the use of fewer approximations. This is true even for complex forms, as long as their complexity is finite. It is especially significant in the limit of the very small scales used in fractal geometry's definitions.[12] On the other hand, non-fractal geometries cannot provide a complete description of infinite complexity, neither locally nor globally. The analysis of fractals by these geometries is, at best, partial or approximate. If exactness and full description are required, non-fractal geometries

[8]For a discussion of the various kinds See Falconer, *Fractal Geometry*; Peitgen and Saupe, *The Science of Fractal Images*; and Mandelbrot, *The Fractal Geometry of Nature*.

[9]The concept of a detail is not primitive, and seems reducible to concepts of non-fractal geometries. This needs a more rigorous discussion than the present one.

[10]In choosing this definition I generally agree with Mandelbrot. It is equivalent to his definition of a fractal, as a form having fractal dimensions greater than its topological dimensions; see Mandelbrot, *The Fractal Geometry of Nature*, pp. 15 and 361ff. I find the term 'infinite complexity' simply more indicative of the fractal's nature. For a presentation and explanation of various kinds of fractal dimensions see Falconer, *Fractal Geometry*, pp. 25–67.

[11]For example, Falconer in *Fractal Geometry* chooses to define fractal as a Wittgensteinian family concept; see pp. XIII–XXI.

[12]Especially in the definitions of the various fractal dimensions; see Falconer, *Fractal Geometry*, pp. 25–67.

can only be used to analyze finitely complex forms. For these reasons, I prefer a definition of fractals that includes all and only the forms not analyzable by non-fractal geometries.[13] For this purpose, infinite complexity seems to serve well.

Equipped with this definition, we can return to the difficulties posed by the concept of fractal. *Prima facie*, if a form has infinitely many details, its very specification is impossible, either as numerical data or as a geometrical entity presented graphically. Fortunately, certain kinds of infinite complexity can be generated by finitely long algorithms, including loops that run infinitely many times, constantly generating additional numerical data. These data can be presented graphically, with the aid of computerized graphical equipment.

Practically, of course, only the finite amount of data obtained after a finite time can be presented. An image obtained this way is not a fractal, but an intermediate stage on the infinite route to the fractal. This is the case whether the process is viewed as constructing the fractal or as discovering it. The infinitely complex fractal itself cannot be obtained, since it is generated by an essentially infinite process. We finite creatures can only examine images that are intermediate stages of this creation, but these images are necessarily of finite complexity, and therefore are by the above definition not fractals.

Fractals seem analogous to irrational numbers in their ontic and epistemic essence. Both can be described as generated by essentially infinite processes. Only their 'heads' can be obtained, but these alone are of a different category from the whole, being finitely complex forms or rational numbers, respectively. The epistemic and ontic status of the 'tails' is a problem at the core of the realism versus anti-realism debate. One can refer to an irrational number by mentioning its name, such as '$\sqrt{2}$', 'π' or 'e', and in this sense one relates to a certain entity. On the other hand, these entities are generated by essentially infinite algorithms. There are two kinds of such infinite algorithms. Some of the irrational numbers are generated by algorithms having finite descriptions, and containing loops that are repeated endlessly. These irrationals can be referred to through descriptions of their generating algorithms, though not directly through specifications of their numerals. Other irrational numbers are composed of an infinitely long random series of numerals. This subset is the larger one. By definition, these numbers are generated by algorithms no shorter than their endless series of numerals. Therefore, they cannot be specified even by descriptions of their generating algorithms. There is no way whatsoever to refer to such numbers. For a Mathematical Platonist, these facts present no difficulty, since an irrational number exists whether it can be specified or not. The inability to specify a number amounts to a difficulty of reference, at best. For a Constructivist, the inability to construct an irrational number presents an ontological and epistemological difficulty.

[13]This definition is not circular, although it may seem so at first sight. Agreed, fractal geometry was developed for the purpose of analyzing forms that were known beforehand (though not by the name 'fractals', coined by Mandelbrot). But, once the geometry is given, its properties can be used for the definition of its proper subject matter.

An inconstructible number does not fully exist. Only its constructible 'head' exists, but the non-complete output of the infinite algorithm is a rational number, not an irrational one.

The analogy with fractals is illuminating. On the one hand, one can refer to certain fractals by mentioning their names, such as 'The Mandelbrot Set' or 'The Von Koch Curve'. On the other hand, the numerical data that describe such entities are generated by algorithms that are by definition infinite. Here, too, two subsets can be discerned. One includes fractals generated by algorithms having finite descriptions, such as the above Mandelbrot and Von Koch fractals. The other includes random fractals, whose infinite complexity can only be described by specifying each and every detail. By the same logic used with regard to irrational numbers, this second subset is the larger one. As in the case of the irrationals, fractals of the first subset can be referred to through descriptions of their generating algorithms, although they cannot be described directly. Fractals of the random subset cannot be referred to even in this indirect mode. It turns out, then, that the ontological and epistemological debates and difficulties regarding irrational numbers are also relevant for and applicable to fractals.[14]

Fractals present, however, an additional problem not encountered when discussing irrational numbers. It is the fact that a fractal is, first and foremost, a geometrical object. True, geometrical objects can be constructed using numerical algorithms and data, but they are nevertheless different entities. A geometrical object is embedded in space, be it real or imaginary, Euclidean or other, and of any number of dimensions. As such it is thought of as existing all at once. Irrational numbers can be thought of as existing all at once when presented geometrically and not numerically; in fact, the irrationality of the square root of two was discovered by the Greeks via its presentation as the diagonal of a unit square. However, when examining fractals one faces an entity that is, on the one hand, a geometrical object, and, on the other hand, is generated by an essentially unending process. There is no way to present it all at once analogous to the geometrical presentation of the irrational $\sqrt{2}$. For the Mathematical Platonist, these facts present no difficulties. Fractals exist all at once in the realm of ideas. Difficulties are encounted only by a Constructivist. From a Constructivist point of view, a fractal is a constantly created, forever dynamical geometrical object. It may be described as a *geometrical process*, rather than as a geometrical object. Only intermediate stages on the way to a fractal can be said to exist all at once, but these, as I have already said, are not fractals.[15]

These considerations lead to the conclusion that fractals cannot exist at any actual time and place, other than in a Platonist realm of ideas. In light of the elusive nature of fractals, it is already hard to understand why one might say that natural objects

[14]For example, Roger Penrose uses the Mandelbrot set to illustrate his argument for Mathematical Platonism. See R. Penrose (1989) *The Emperor's New Mind* (New York: Oxford University Press), pp. 98–104 and 123–128.

[15]The advantage of the Platonist with regard to the status of fractals can be used as an argument for Platonism. However this is *not* Penrose's argument in *The Emperor's New Mind*.

are fractals, other than as a very general approximation. The proper application or scope of such an approximation will be discussed later.

3. Empirically, Natural Objects Are Not Fractals

Indeed, I shall now argue that, as a matter of empirical fact, there is no ground to say that natural objects have a fractal geometry. Mandelbrot's hypothesis rests on the fact that the so-called fractal images *prima facie* seem to imitate natural forms. However, close examination reveals that this *prima facie* resemblance is both superficial and misleading. Superficial, since in most of the cases the images turn out to be unnatural. Misleading, since in other cases the success in imitating nature is better explained without resorting to the fractal geometry, and therefore has nothing to do with fractals. For this reason I do not refer to these images as 'fractal images', but as 'so-called fractal images'.

Let us now turn to examine why so-called fractal images seem to imitate nature. When one says that such a form resembles a natural form, one does not relate to the infinitely remote output of the fractal's algorithm, but to the image in front of one's eyes. That image is not a fractal, for it is of a finite complexity. It is an intermediate stage on the way to a fractal. There are several families of fractals, characterized by the kinds of algorithms used to generate them. The intermediate stages of each such family seem to imitate a different kind of natural form. The explanation for the imitations' success is different for each family, because it relates to the algorithm for its generation.

3.1. Forms Mistakenly Classified As Fractals

Two kinds of forms, that are not fractals, were for some reason mistakenly classified as such. The first type is that of the L-systems. These are models of plant development, and thus are also used for synthesizing realistic images of plants.[16] A variation of these systems can be used as an algorithm that generates fractals of the self-affine kind (to be discussed later). This fact created a confusion even among experts, who somehow started to see the other, clearly non-fractal products of L-systems, as fractals also.[17] The other family of forms is the DLA kind (Diffusion Limited Aggregation). This method is successfully used to simulate phenomena of aggregation, from electrolysis to moth growth.[18] The definition of the forms that this method generates includes a pixel of a finite size, a geometrical atom. Therefore, their complexity can be increased only by expansion. But, within a given volume of their embedding space their

[16]See P. Prusinkiewicz and J. Hanan (1989) *Lindenmayer Systems, Fractals and Plants, Lecture Notes in Biomathematics* (New York: Springer).

[17]See Peitgen and Saupe, *The Science of Fractal Images*, plates 16c and 16d after p. 114.

[18]For a model of electrolysis see Falconer, *Fractal Geometry*, pp. 267–272. Moth growth can be seen in Peitgen and Saupe, *The Science of Fractal Images*, p. 38.

complexity is essentially finite, contrary to the definition proposed above. Therefore, they are not fractals.

3.2. *Unnatural Fractals That Are Embedded In Abstract Non-isotropic Spaces*

Complex iterations forms constitute a third kind of so-called fractal images. To this family belongs the famous Mandelbrot set. Using this method, forms *prima facie* reminiscent of microorganisms were generated.[19] However, these forms are embedded in the non-isotropic complex space. Saying that they can be used to explain life forms amounts to holding the radical belief that, with regard to such forms, physical space is non-isotropic. However, the assumption that physical space is isotropic is otherwise overwhelmingly corroborated. Furthermore, close inspection or a simulated magnification of what superficially seems like organs, reveals characteristics of Julia sets. The resemblance of the latter to life forms is questionable. It may be that psychological Gestalt effects in the observer force the intricate details into the known patterns of organic forms.[20] The Julia and Mandelbrot sets of complex iterations are also used to generate fantastic landscapes.[21] The non-isotropic complex space considerations are relevant here, too.

The strange attractors fractals are Poincaré sections of the phase space trajectories of dynamic chaotic systems. Their appearance *prima facie* resembles mixing fluids. This impression is strengthened when one notices that real fluids' images also depict sections or surfaces. However, no far-reaching physical conclusions can be drawn from this resemblance, as the attractors are embedded in the non-isotropic phase space of position and momentum, not in the isotropic physical space. Incidentally, strange attractors are known as embodying the fractal nature of the initial conditions of chaotic systems. This belongs to the dynamical or temporal part of Mandelbrot's hypothesis, which is not discussed here.

3.3. *Images That Resemble Nature Because They Are Finitely Complex Non-fractals*

Some of the so-called fractal images do seem to resemble natural forms, and quite closely. I shall presently argue that these images resemble nature due to certain characteristics of theirs, characteristics that have nothing to do with fractals or with the fact that they can be viewed as intermediate stages on the way to fractals. In fact, they resemble nature just because they are *finitely* complex, and therefore just because they are *not* fractals. In other words: these images resemble nature, not because they are stages on the way to fractals, but rather *in spite* of this fact.

[19]See C. A. Pickover (1986) 'Biomorphs: Computer Displays of Biological Forms Generated From Mathematical Feedback Loops', *Computer Graphics Forum* **5**, 313, and A. K. Dewdney (1989) 'Computer recreations', *Scientific American* July, 92.

[20]Compare the different scales of the pictures in pages 314 and 315 in Pickover. 'Biomorphs: Computer Displays of Biological Forms Generated From Mathematical Feedback Loops'.

[21]See Peitgen and Saupe, *The Science of Fractal Images*, covers and colored plates after p. 114.

The first kind of such images are self-affine forms.[22] Many plants and crystals are self-affine (or self-similar) in a finite number of scales. There is no wonder, then, that images revealing a finite self-affinity resemble these natural forms. However, if one continues the algorithm that generates self-affinity, wishing to approach the self-affine fractal by generating self-affinity in numerous scales, the obtained image ceases to resemble the natural one. The reason seems trivial, and would not have required any detailed discussion, were it not for the widespread false idea that these images resemble nature *due* to their being stages on the way to a fractal, rather than *in spite* of this fact. Let us then discuss this matter briefly, by examining an example.

A paradigmatic example of self-affinity is the image of a fern. A small number of self-affine steps generates a persuasive image of a fern.[23] However, as one expects the self-affinity to end at the scale of the smallest leaves, additional self-affine details spoil the natural impression. In order to compare the real fern and the fractal in smaller scales, magnification is needed. A magnification of a fractal is not done with the aid of a magnifying glass, since such an action would reveal the details of the paper or monitor on which the fractal is plotted. The substitute is a simulated magnification, done by plotting a portion of the original image on the whole page. Were the result a graphical representation of a natural form, the simulated magnification would have had the effect of a real magnification, of observing a fern with a magnifying glass, or of approaching the object. *But it does not.*[24] With regard to natural ferns, one can easily tell the scale that one is observing, with possible errors ranging between the few scales in which the fern is self-affine. But with regard to the fractal fern this is not possible, as all scales appear the same. One cannot tell whether one is observing the scale of one centimeter or of one Angstrom. True, such is the effect of self-affinity. But, it is not a natural effect. Therefore, the fact that the so-called fractal images resemble ferns is a result of their *finite* complexity, of the fact that they are self-affine in only a finite number of scales. In other words, they resemble natural forms due to their *not* being fractals.

This fact has surely not escaped the eyes of Mandelbrot and the proponents of his hypothesis. However it seems not to have belittled the role they give to fractal geometry in describing nature. There lies the mistake. Forms that can be described by non-fractal geometries at all, are more exactly described by them. Therefore a physics of such forms should be based on their non-fractal geometrical description. The proper role of the fractal geometry, in approximating natural forms, will be discussed later.

The non-natural characteristics of self-affine images, encountered in simulated magnifications, are met also in images of the statistically self-similar kind of fractals.

[22]For a discussion of self affine fractals, see Falconer, *Fractal Geometry*, pp. 113–137.
[23]For example, see Peitgen and Saupe, *The Science of Fractal Images*, plate 32 after p. 114.
[24]See Peitgen and Saupe, *The Science of Fractal Images*, p. 239.

These images are used to depict landscapes.[25] In each of the images, be they of coastlines, mountains, or whole planets, not more than a few scales are effectively visible. The largest visible scale is statistically ineffective as one can see only a small portion of it, such as a single mountain. The medium scales, of which only a few are practical, are effectively and clearly visible, and determine the general appearance of the image. The yet smaller scales are not visible, and therefore have no effect on the depicted landscape's appearance. Now saying that natural landscapes are statistically self-similar amounts to saying that, *in nature*, the invisible scales are statistically self-similar to the visible ones. However, empirically this is not the case. Again, the reason seems trivial, but the widespread belief to the contrary necessitates a brief discussion.

The creator of a statistically self-similar image chooses the right fractal dimension in order to generate a realistic impression. Actually, the choice relates mainly to the few medium scales, the only ones that are effective in creating the general impression of the image.[26] Now, imagine that one obtains a realistic landscape, and then simulates its magnification. Assuming that nature is statistically self-similar, one expects to experience the effect of approaching the depicted landscape. *But this does not happen.* After a simulated enlargement by 10^7, equivalent to approaching from ten kilometers in the air to one millimeter off the ground, the landscape looks as if it is seen from the initial altitude.[27] True, again, this is an expected effect of good statistical self-similarity, but it is not the expected effect of simulated enlargements of *natural* landscapes. In reality one can usually quite successfully guess one's altitude, using the non-similarity of the different scales.[28] The conclusion is, that the landscape images resemble nature only in a finite number of scales, and therefore just because they are *finitely* statistically self-similar. In other words, they resemble natural forms just because they are *not* fractals.

Summing up the explanations for the success in imitating nature, the finitely self-affine and statistically self-similar forms resemble some scales of natural forms due to their finite complexity, i.e. due to their *not* being fractals. Other so-called fractal images are either embedded in essentially non-isotropic spaces, or do not resemble

[25]Examples of such landscapes can be found in Peitgen and Saupe, *The Science of Fractal Images*, front and back covers and colored plates after p. 114, and J. Feder (1988) *Fractals* (New York: Plenum Press), colored plates. For a discussion of statistically self similar algorithms that can generate fractals, see Falconer, *Fractal Geometry*, pp. 224–253, and Mandelbrot, *The Fractal Geometry of Nature*, pp. 12–13 and 200–276.

[26]For example, in Mandelbrot, *The Fractal Geometry of Nature*, colored plate at p. C9, the 'Moon' is of fractal dimension 2.2 and the Earth of 2.5. The effect and the process of choosing the right dimension can be seen at Peitgen and Saupe, *The Science of Fractal Images*, colored plates 11–13, and Mandelbrot, *The Fractal Geometry of Nature*, pp. 264–267 and plate C14. The effect of varying the random function is exemplified in Peitgen and Saupe, *The Science of Fractal Images*, colored plates 8–10.

[27]See Peitgen and Saupe, *The Science of Fractal Images*, p. 23.

[28]The photographs in the final section of Mandelbrot, *The Fractal Geometry of Nature*, confirm this argument; different scales are statistically non-self-similar in those pictures.

nature at all, or are not fractals in the first place. Thus, the empirical data disconfirms Mandelbrot's hypothesis.

4. The Use of Fractal Geometry As a Rough Approximation in the Analysis of Natural Forms

Agreed, the fact that no natural form is actually a fractal need not prevent one from using fractal geometry whenever it turns out to be useful. One might raise the objection that so-called fractal images, being intermediate stages on the route to fractals, are approximations of fractals, in the same sense that a drawn line is an approximation of the ideal one-dimensional line. By stating that the approximation is qualitatively different from the approximated, I have only repeated an idea well known in non-fractal geometries. The objection is, why stress this point? Why not call the approximated fractal 'fractal' in the same sense that one calls an approximated line 'line', while remembering the approximation at the back of one's mind? In order to answer this objection, let us see how such an approximation can be carried out.

Geometrical forms can be ascribed to physical objects by mapping each point of the physical space to a point of an abstract space having some suitable and convenient geometry and a coordinate system. A natural form is a geometrical form in the chosen abstract space that conforms to a physical object by the said mapping.[29] Accordingly, natural objects can be analyzed using fractal geometry in the following manner. First, the natural object is to be mapped to the abstract space in which it is ascribed a geometrical form. The obtained geometrical form is not a fractal, but a finitely complex form. Infinitely many details are then added to the abstract form in the abstract space. For example, if the original natural object is finitely self affine, its self-affinity is to be continued *ad infinitum*. It must be remembered that the fractal thus obtained is not the natural form of the natural object, but an abstract geometrical form in the abstract space, based on the object and with infinitely many details added. Fractal geometry can be applied to this fractal. The consequences of such an application, such as the calculated fractal dimensions, can be considered to be approximately true of the original, non-fractal form. The circumstances in which such an approximation is appropriate depend on the purpose of the analysis. For instance, it may turn out to be useful when the object's smallest details are smaller than is deemed relevant or measurable. In such cases, the application of fractal geometry can turn out to be convenient, since it enables a description of general traits of the forms without having to measure each and every detail of them.

However, one must bear in mind that fractal geometry can only be approximately applied to natural forms. It can be applied for the sake of convenience in the

[29]This presentation of the concept of a natural form is not without problems, but it suffices presently. See Hans Reichenbach (1957) *The Philosophy of Space and Time*, trans. M. Reichenbach and J. Freund (New York: Dover).

appropriate circumstances, but, strictly speaking, these forms are not fractals. This last point cannot be overemphasized, since neglecting to remember it has actually led to gross misunderstandings with regard to the status of fractal geometry in the scientific analysis of natural forms. It has led to the mistaken opinion that the ability to approximately apply fractal geometry in some cases indicates a deep natural structure, hitherto ignored, and profoundly different from the prevailing theories regarding natural forms. This, in turn, led to the conclusion that fractal geometry is to be the basis of a new physics relating to natural forms. The failure to discern the approximation from the approximated has led to far-reaching conclusions that groundlessly challenge the atomistic paradigm, as we shall shortly see.

My answer to the above objection is that one can call the approximation of a fractal 'a fractal', as long as one abstains from relating infinite complexity to finitely complex forms. It is necessary to insist on discerning between fractals and their approximations exactly because a neglect of this discernment has led to mistaken conclusions with regard to the geometry of nature.

5. The Lack of a Physical Interpretation of Mandelbrot's Hypothesis

Generally speaking, scientific theories connect mathematical functions to nature by interpreting them. An interpretation of a function relates to the function as embodying a law of nature, through its structure and parameters. The interpretation has nothing to do with the shape of the function's plot, as plots heavily depend on the choice of coordinate systems. In the scientific literature, graphical representations of mathematical functions are used only for the sake of clarification or for didactic purposes. A scientist presented with a plot without the function that generates it, can make nothing of the plot unless the function is specified; it is scientifically meaningless. A case close to this is when the scientist is presented with a plot and its generating function, but that function does not belong to any theory. This time, the meaninglessness is due to lack of context and lack of interpretation. The last case is the case of so-called fractal images.

Kadanoff, in the above quotation, is well aware of this fact. He recommends a search for interpretation. In which direction can it be sought? The resemblance between these plots and pictures of natural objects cannot serve as an interpretation of their generating functions, because it does not relate to these functions' structure and parameters. Can an interpretation be found within existing science? The increasing popularity of Mandelbrot's hypothesis regarding natural forms has not been accompanied by serious attempts to make it coherent with scientific theories that are otherwise widely corroborated. Probably not without reason: such a coherence cannot be obtained. Let us briefly discuss this point.

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the *atomic hypothesis* that *all things are made of atoms*.[30]

These words of Richard Feynman embody one of the most basic ideas of modern science. In its original context, at the introduction to *The Feynman Lectures on Physics*, the word 'atoms' is not meant literally, but refers to the objects decomposable into protons, neutrons and electrons. However, if we interpret the word to mean that there are things such that all other things are composed of them, then Feynman's words express the paradigm on which the whole scientific conception of the material world is based. I shall emphasize that the paradigmatic idea consists of the compact statement that elementary particles exist; their number and nature is not part of the paradigm.

Under the atomistic paradigm, the geometrical characteristics of natural forms are determined as follows. First, each elementary particle is mapped to an abstract space, in which it is ascribed a geometrical form, whether as a point or as having some volume, depending on its specific properties.[31] A macroscopic object is an aggregate of elementary particles, and the mapping mirrors their relative positions. The natural form of a large object is, then, an aggregate of the forms ascribed to its elementary particles, with their appropriate arrangements and relative positions.

Fractals are defined above as being infinitely complex, as having an infinite number of details within a finite volume of the embedding space. However, if an object is made of elementary particles, then the number of details its geometrical form has is finite. This number is determined by the number of the elementary particles and their relative positions. It may be very large, but it is always finite. The inevitable conclusion is that according to the modern scientific conception of natural forms, natural objects cannot be fractals. The very idea of an object having an infinite number of details contradicts the atomistic paradigm and therefore is inconsistent with modern science. Mandelbrot's hypothesis cannot then receive an interpretation within known science. It needs to be interpreted within a radically new, non-atomistic science. As the atomistic paradigm is otherwise invariably corroborated, the sole motivation for rejecting it in search of a 'physics of fractals' (in Kadanoff's words) is the *prima facie* resemblance of pictures. This resemblance itself we have shown to be superficial and misleading. It seems that the fact that a satisfactory non-atomistic interpretation was not offered, not even in the most general terms, is not without a reason.

The proponents of Mandelbrot's hypothesis did not mean, perhaps, to reject or challenge the atomistic paradigm, or else they would have provided deeper arguments. Nevertheless, the hypothesis taken by itself implies such a challenge. Lacking interpretation, the status of Mandelbrot's hypothesis is that of a mere

[30]Richard P. Feynman, Robert B. Leighton and Matthew Sands (1963) *The Feynman Lectures on Physics* (Reading, Massachusetts: Addison-Wesley) Vol. 1, pp. 1–2.

[31]Although I cannot defend the point in detail here, I think it is clear that the points in this paragraph and the next are not affected by the differences between the ontologies offered by the various interpretations of quantum theory.

collection of abstract functions, no different from any other such collection. It has no scientific status, not even the status of a candidate for becoming a scientific theory.

6. Conclusions

The impressive so-called fractal images offered a great temptation to science. On the one hand, while science has it that natural forms are in principle reducible to the structure and processes at the level of elementary particles, the computational difficulties make such a reduction impossible. Thus science lacks a practical theory of macroscopic forms, in all but a few simple cases. On the other hand, successful imitation of natural forms seemed attainable by using simple algorithms. The temptation to see in these images a thread that can lead to a theory of macroscopic forms was so great, that basic principles of scientific methodology seem to have been forgotten.

The first forgotten principle is the need to have at least an approximate empirical agreement between the hypothesis' predictions and reality. The experiment of comparing imitated magnifications of fractal images and real magnifications of natural forms conspicuously reveals their deep differences. This has not led to abandoning the fractal structure hypothesis. It has not led to restricting the application or scope of fractal geometry to just approximations.

The second forgotten principle is the need for consistency between various scientific theories. The fact that the concept of a fractal object is profoundly inconsistent with one of the most basic postulates of modern science, the atomistic paradigm, did not lead to emphasizing the restricted role and approximative nature of fractal geometry in the analysis of natural forms. Instead, scientists and non-scientists alike advocate the view that this geometry offers a novel method of analyzing nature that reveals a deep structure hitherto ignored. This radical view was not supported by any argument other than the superficial resemblance of pictures.

The third forgotten principle concerns the difference between a system and its approximation or idealization. Whenever the two are so qualitatively different that they seem governed by different scientific paradigms, the paradigm relating to the real system is of course to be preferred to that relating to the approximation.

Kadanoff, in the quotation in Section 1, is aware of the problem that a superficial resemblance is far from being enough, and that unless the fractal functions receive a proper interpretation they cannot be said to have a scientific meaning. He recommends the creation of a physical theory within which such functions will be meaningful. This actually implies the search for a radical theory not based on the atomistic paradigm. However, Kadanoff's very recommendation rests on the assumption that forms of natural objects can be successfully imitated by fractals, an assumption that is empirically wrong. A physics of fractals cannot then be born, because fractal geometry is not the geometry of nature.

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